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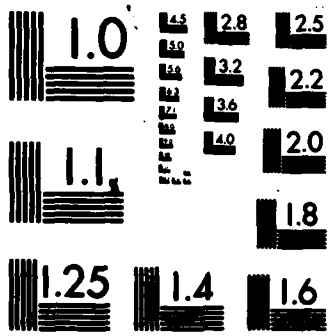

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NAVAL CIVIL ENGINEERING LABORATORY
Port Hueneme, California

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FINITE ELEMENT ANALYSIS FOR COHESIVE SOIL, STRESS AND CONSOLIDATION PROBLEMS USING BOUNDING SURFACE PLASTICITY THEORY

December 1983

An Investigation Conducted by
UNIVERSITY OF CALIFORNIA, DAVIS

N62583-83-M-T062

METRIC CONVERSION FACTORS

Approximate Conversions to Metric Measures

Symbol	When You Know	Multiply by	To Find	Symbol
		LENGTH		
in	inches	*2.5	centimeters	cm
ft	feet	30	centimeters	cm
yd	yards	0.9	meters	m
mi	miles	1.6	kilometers	km
		AREA		
in^2	square inches	6.5	square centimeters	cm^2
ft^2	square feet	0.09	square meters	m^2
yd^2	square yards	0.8	square meters	m^2
mi^2	square miles	2.6	square kilometers	km^2
	acres	0.4	hectares	ha
		MASS (weight)		
oz	ounces	28	grams	g
lb	pounds	0.45	kilograms	kg
	short tons (2,000 lb)	0.9	tonnes	t
		VOLUME		
tsp	teaspoons	5	milliliters	ml
Tbsp	tablespoons	15	milliliters	ml
fl oz	fluid ounces	30	milliliters	ml
c	cups	0.24	liters	l
pt	pints	0.47	liters	l
qt	quarts	0.95	liters	l
gal	gallons	3.8	liters	l
ft^3	cubic feet	0.03	cubic meters	m^3
yd^3	cubic yards	0.76	cubic meters	m^3
		TEMPERATURE (exact)		
°F	Fahrenheit temperature	5/9 (after subtracting 32)	Celsius temperature	°C

*1 in = 2.54 (exactly). For other exact conversions and more detailed tables, see NBS Misc. Publ. 286, Units of Weights and Measures, Price $2.25, SD Catalog No. C13.10:286.

Approximate Conversions from Metric Measures

Symbol	When You Know	Multiply by	To Find	Symbol
		LENGTH		
mm	millimeters	0.04	inches	in
cm	centimeters	0.4	inches	in
m	meters	3.3	feet	ft
m	meters	1.1	yards	yd
km	kilometers	0.6	miles	mi
		AREA		
cm^2	square centimeters	0.16	square inches	in^2
m^2	square meters	1.2	square yards	yd^2
km^2	square kilometers	0.4	square miles	mi^2
ha	hectares (10,000 m^2)	2.5	acres	
		MASS (weight)		
g	grams	0.035	ounces	oz
kg	kilograms	2.2	pounds	lb
t	tonnes (1,000 kg)	1.1	short tons	
		VOLUME		
ml	milliliters	0.03	fluid ounces	fl oz
l	liters	2.1	pints	pt
l	liters	1.06	quarts	qt
l	liters	0.26	gallons	gal
m^3	cubic meters	35	cubic feet	ft^3
m^3	cubic meters	1.3	cubic yards	yd^3
		TEMPERATURE (exact)		
°C	Celsius temperature	9/5 (then add 32)	Fahrenheit temperature	°F

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The equations governing the consolidation, and the stress and strains states for soil structures are reviewed and their historical development is discussed. Numerical analysis concepts are used to express these equations in incremental form. A variational statement of these incremental equations is formulated and used in the development of a comprehensive

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finite element analysis. The concepts used in developing the variational statement are somewhat different from those used by most other investigators and appear to offer certain advantages for inelastic formulations. Finally results obtained with the finite element analysis are compared to known solutions with good results.

For the convenience of the reader, the total report on the project is presented in four parts. As noted above a description of the consolidation theory and certain theoretical features of the finite element analysis are described in the body of the main report (CR 84.006). The second part (CR 84.007) describes the numerical evaluation of the incremental form of the bounding surface model. Finally "user's manuals" for the 2-D and 3-D finite element programs are given in two additional reports (CR 84.008 and CR 84.009).

Justification

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1. SCOPE OF PROJECT

The goal of the project is to develop two special purpose finite element codes for the analysis of cohesive-soil, stress and deformation problems including consolidation effects. Specifically the codes are to make use of the new comprehensive, bounding surface plasticity constitutive model for cohesive soils [1,2,3].

2. INTRODUCTION

The analysis is limited to small deformations and displacements, and classical consolidation theory. In addition, it is restricted to two ideal conditions of saturation. The first is when the soil is completely saturated and consideration is given to the development and dissipation (due to water flow) of excess pore water pressure; included in this case are ideal undrained conditions. The second case assumes that the degree of saturation is sufficiently small (or ideal drained conditions exist) that the pore water pressure is zero, and the presence of water can be completely accounted for by the increased unit weight of the soil.

Consolidation theory for saturated soils is well established and can be found in a number of references [e.g., 4-8]. The form of the theory used in this work is taken from [8] with only slight modification and some changes in notation; the theory is summarized in a later section.

A number of finite element analyses have been developed [6-12] for soil consolidation problems; most have been limited to linear elastic material behavior which is unrealistic for cohesive soils and none have used the newly developed bounding surface plasticity theory. Because the finite element concepts employed in the programs are standard, the section describing the analysis will be brief in nature.

3. A BRIEF LITERATURE REVIEW OF THE THEORY AND FINITE ELEMENT APPLICATIONS FOR CONSOLIDATION

This section reviews the theory of consolidation and examines several representative Finite Element analyses for its evaluation. It is not an exhaustive catalogue of research in consolidation theory and analysis, but instead attempts to demonstrate some of the advantages and the difficulties present in Finite Element models.

3.1 Consolidation Theory

The birth of Soil Mechanics as a modern engineering discipline occurred in the 1920's, when the Austrian engineer Karl Terzaghi proposed his theory of the consolidation of saturated fine-grained soils under applied loads [13]. Terzaghi had been studying the phenomenon of the reduction in void space of soils underlying foundations. He correctly perceived that the time-dependent settlement from consolidation of these soils was due to the flow of water out of the soil skeleton as the voids decreased in size. The permeability of the soil dictates the rate at which these movements take place. The soil skeleton thus acts like a large sponge in response to an applied load.

Many of the most important features of consolidation can be motivated by considering a greatly simplified mechanical model, the 'spring analogy', Fig. 1.

Consider a cylinder which contains a piston, valve, and elastic spring, Fig. 1a. This cylinder is filled with an incompressible fluid. If a force is applied to the piston with the valve open, the force is initially carried by the fluid, Fig. 1b. With time, however, the fluid drains from the cylinder under the applied force, and more of the force is carried by the spring, Fig. 1c. Finally, a new equilibrium position is reached, Fig. 1d, where all of the force is carried by the spring, and the excess fluid pressure drops to zero.

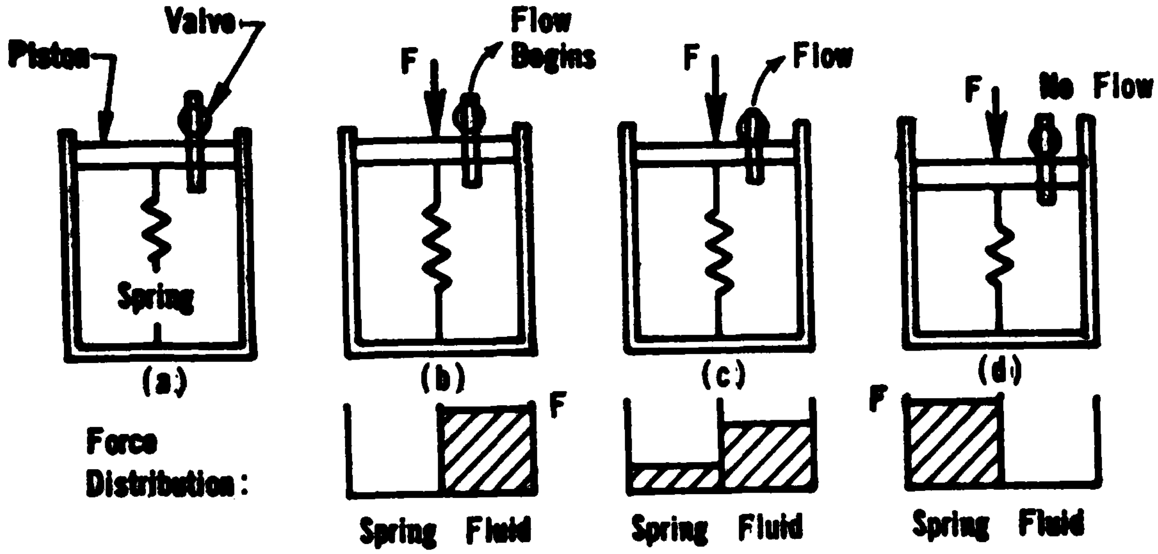

Figure 1. Spring Analogy For Consolidation

This simple analogy gives an accurate model of consolidation of saturated soils. The spring represents the compressible soil skeleton, the fluid represents the pore water that fills the soil voids, and the valve represents the permeability of the soil. It is easily seen that the rate of deformation depends upon the soil's permeability (i.e. how much the 'valve' is opened). For coarse-grained soils (sands, gravels) the permeability is so high that the deformation response is essentially instantaneous. Thus, time-dependent consolidation effects are generally observed only in fine-grained soils (silts, clays). The entire process can be made precise by introducing the following stress concepts (here, in contrast to later equations, compression is taken as positive):

τ : the total stress due to the applied load

σ': that portion of τ carried by the soil skeleton

p : that portion of τ carried by the pore water

At any time after the load is applied:

$$\tau = \sigma' + p \tag{1}$$

Terzaghi realized that the deformation of the soil depended directly upon σ' and not τ. He called σ' the effective stress, and the excess pore pressure p the neutral stress (since it does not directly affect the soil's deformation). This concept of effective stress is central to the study of soil mechanics, whether consolidation is present or not.

Terzaghi developed a simple model [14] for consolidation of soil layers, subject to the following assumptions, see Fig. 2.

1. The consolidating layer is horizontal, of infinite extent (laterally) and of constant thickness h.

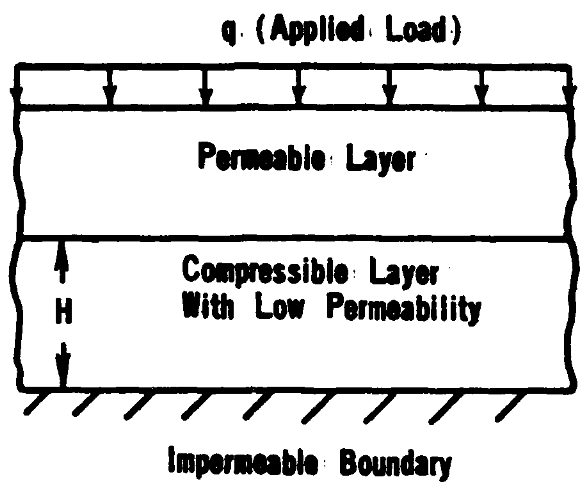

Figure 2. Geometry of Terzaghi's Model

2. The permeability coefficient (k) and volume compressibility (m_v) are constant throughout space and time. The volume compressibility m_v represents the ratio of volumetric strain to applied effective stress. Since the consolidating layer is of infinite lateral extent, the vertical strain is the volumetric strain.
3. The pore water drains only in the vertical (z) direction.
4. The time rate of compression depends only upon low soil permeability: visco-elastic properties of the soil skeleton are not considered.
5. The fluid obeys Darcy's Law: flow is proportional to the gradient of pore water pressure.

$$v = - \frac{k}{\gamma_w} p,_i \tag{2}$$

6. Strains are small compared to unity.
7. The applied load τ is constant for all time. Thus, since τ is given, if the pore pressure p is known, so is the effective stress σ', Eq. (1).

With t ese assumptions, Terzaghi derived a differential equation for this one dimensional case:

$$\dot{p} = \frac{k}{\gamma_w m_v} p,_{ii} \qquad i=3 \tag{3}$$

This equation is identical to the heat, or diffusion equation, and it can be solved using separation of variables for various boundary conditions, Fig. 3. Unfortunately, many of the assumptions made are unrealistic enough to warrant a more general theory. In particular, material properties are not constant, and consolidating layers are not of uniform width or of infinite lateral extent. However, Terzaghi's theory has been widely used in estimating foundation settlements and consolidation rates.

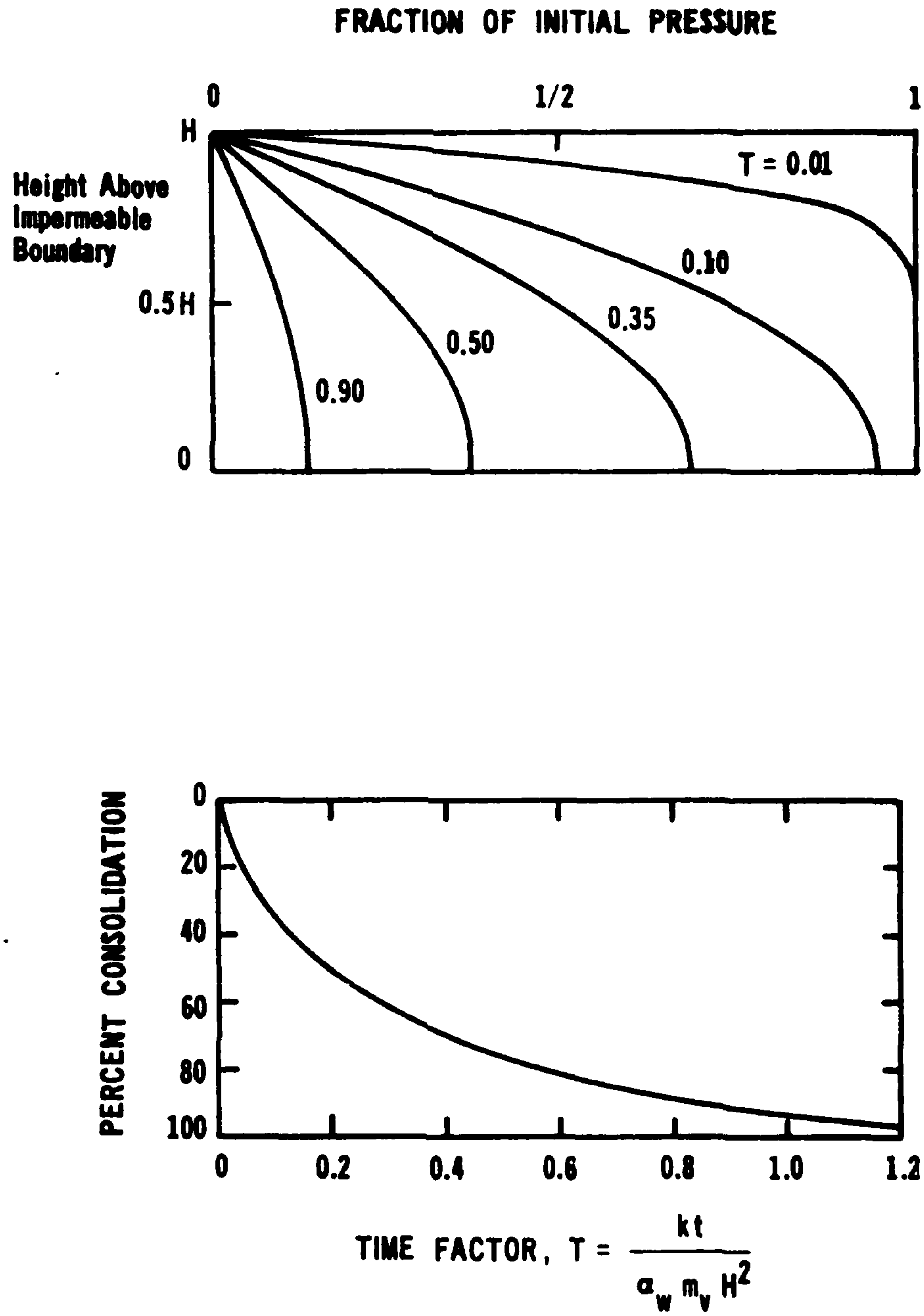

Figure 3. Excess Pore Pressure and Average Consolidation Ratio

In 1941, Biot [4] extended Terzaghi's consolidation theory to the general three-dimensional case, and considered loads that varied with time. If this extended theory is restricted to fully saturated soils where the applied load is constant over time, the generalized model is governed by the same equation as the generalized heat (or diffusion) problem:

$$\dot{p} = \frac{k}{\gamma_w m_v} p,_{ii} \qquad \text{(sum over } i\text{)} \tag{4}$$

As in the one-dimensional case, this equation can be solved using separation of variables for simple geometries and boundary conditions. In later papers [5,15,16,17], Biot extended his consolidation theory to include effects of anisotropy, inhomogeneity, and more general boundary conditions. (Unfortunately, his choice of notation and physical constants underwent a number of changes. For simplicity and consistency in later comparisons with finite element solutions, Biot's results are paraphrased in the following development).

The resulting system of differential equations for elastic consolidation of an anisotropic soil mass can be summarized as follows:

u_i : soil displacement vector

ϵ_{ij} : strain tensor (tension positive)

σ'_{ij} : effective stress tensor (tension positive)

F_i : body force vector (per unit mass)

k_{ij} : permeability coefficient tensor

E_{ijkl} : elasticity tensor in terms of effective stress

δ_{ij} : Kronecker delta

p_i : pore water pressure (compression positive)

v_i : Darcy velocity

Strain-Displacement Relations:

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \tag{5}$$

Equilibrium Equations:

$$(\sigma_{ij} - \delta_{ij}\, p)_{,j} + \rho_s F_i = 0 \tag{6}$$

Effective Stress-Strain Relation:

$$\sigma_{ij} = E_{ijk\ell}\, \varepsilon_{k\ell} \tag{7}$$

Darcy's Law:

$$v_i + \frac{1}{\gamma_w}(k_{ij}\, p_{,j}) = 0 \tag{8}$$

Equation of Continuity:

$$\dot{\varepsilon}_{ii} + v_{i,j} = 0 \tag{9}$$

Biot solved these equations for such three dimensional cases as a rectangular load distribution, Fig. 4, and a soil with an impervious top layer. He also introduced analytic techniques for solution of a variety of consolidation problems, modelling the consolidating layer as an elastic, semi-infinite half space. Although the mathematical effort required to solve these equations is formidable, the results are of limited practical importance, since assuming infinite depth for a soil layer can lead to serious overestimates of total and differential settlements. The utility of Biot's work lies not in his analytical solutions, but in the development of a fairly general three dimensional theory of consolidation.

3.2 Some Finite Element Models for Consolidation

One of the main reasons for the widespread use of finite element models in mechanics is that the method can be used on problems with complex geometries,

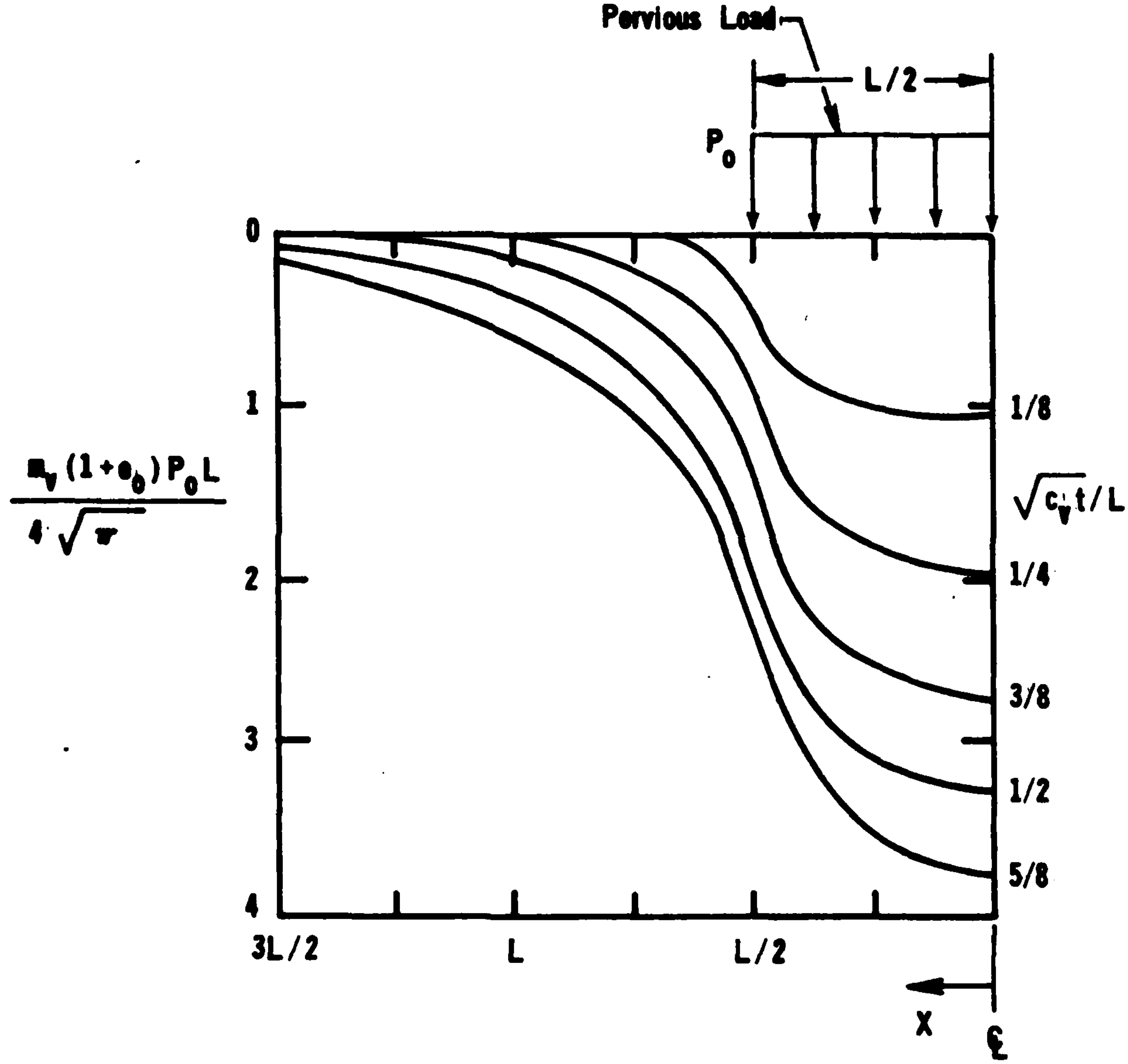

Figure 4. Deformation Under a Rectangular Strip Load (Plane Strain)

variable boundary conditions, and non-homogeneous material properties. Since these difficulties prevent analytic solution of Biot's consolidation equations for many useful problems, finite element procedures have become an important tool in modelling soil consolidation behavior.

Sandhu and Wilson [6] published one of the first finite element models for soil consolidation in 1969. This model combined two existing finite element models, one for the plane strain structural problem, and the second for the solution of the two-dimensional diffusion equation, see Eq. (4). A generalization of Biot's statement of Darcy's Law was used, which included the effect of body forces on the pore water:

ρ_w : water density

k_{ij} : permeability coefficient tensor

$$v_i + k_{ij}(p,_j - \rho_w F_j) = 0 \tag{10}$$

Sandhu and Wilson used a variational approach for the derivation of element equations, combining functionals corresponding to the two coupled problems:

$$G(t) = \int_V \left\{ \frac{1}{2} \sigma_{ij} * \epsilon_{ij} - 2p * u_{i,i} + g' * v_i * p,_j \right.$$

$$\left. - 2\rho_s F_i * u_i + g' * p,_i * \rho_w F_j \right\} dV \tag{11}$$

where $g' = 1$ and $a*b = \int_0^t a(t)\, b(t-s)\, ds$

In order to accomodate traction and flow boundary conditions (natural boundary conditions for the two coupled problems), the following terms must be added to the functional G(t) to obtain the desired functional F(t) for the problem:

T_i : components of prescribed traction vector

S_1 : portion of boundary where traction is prescribed

Q : prescribed normal flux on boundary

S_2 : portion of boundary where flux is prescribed

n_i : direction cosines of outward normal

$$F(t) = G(t) - 2 \int_{S_1} T_i * u_i \, ds_1 - 2 \int_{S_2} g' * Q * p \, ds_2 \tag{12}$$

where $T_i = (\sigma_{ij} - \delta_{ij} p) n_j$ on S_1 $\quad Q = v_i n_i$ on S_2

Sandhu and Wilson evaluated the convolution integrals using a simple two point forward difference formula to obtain a fully explicit time marching scheme. A mixed interpolation model was used on the displacement and pore pressure unknowns: displacements were interpolated using quadratic shape functions, and linear shape functions were used to interpolate the pressure unknowns. Triangular elements were used to define the mesh: therefore the elements incorporated a six node linear strain triangle for the structural (displacement) problem and a three node linear triangle for the fluid problem (pore pressure). This gives a total of fifteen degrees of freedom for each element.

Sandhu and Wilson applied the finite element model to two problems: Terzaghi's one-dimensional soil column and Biot's rectangular strip load on an elastic half-space. In both problems, excellent agreement between the finite element model and the analytic solutions was obtained, see Figs. 5 and 6. Some discrepancy can be seen in the comparison with Biot's strip load solution, but these differences can be attributed to the fact that the finite element mesh (like the soil layers being modelled) is of limited extent, unlike Biot's elastic half space.

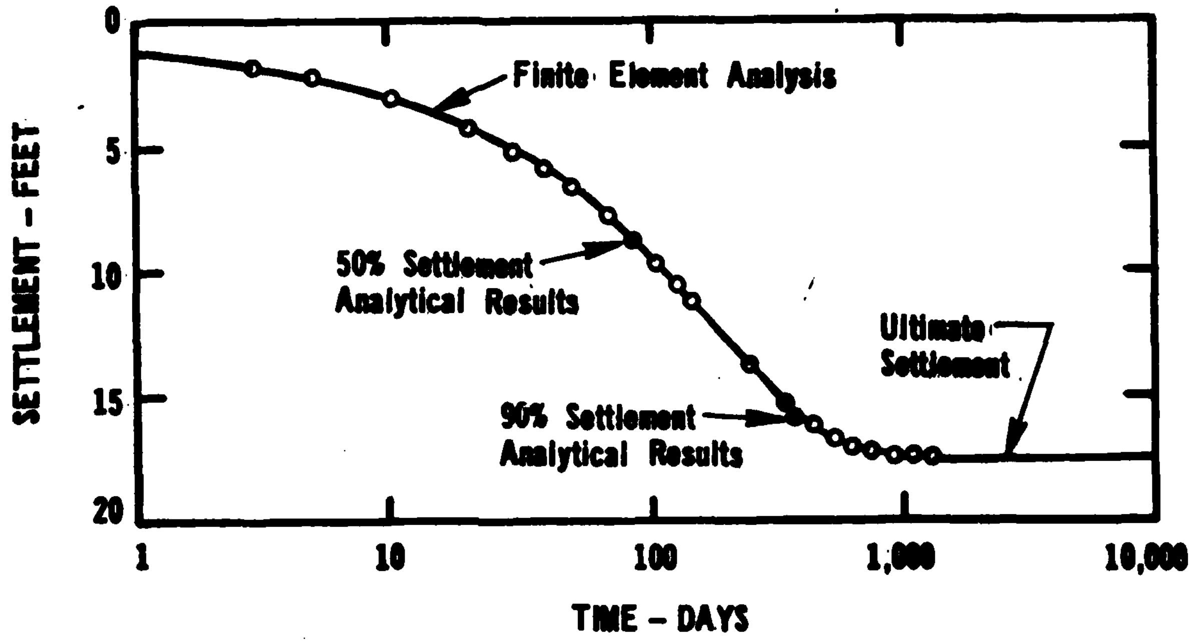

Figure 5. Results of Sandhu and Wilson's Model:
Terzaghi's Problem-Average Consolidation [6]

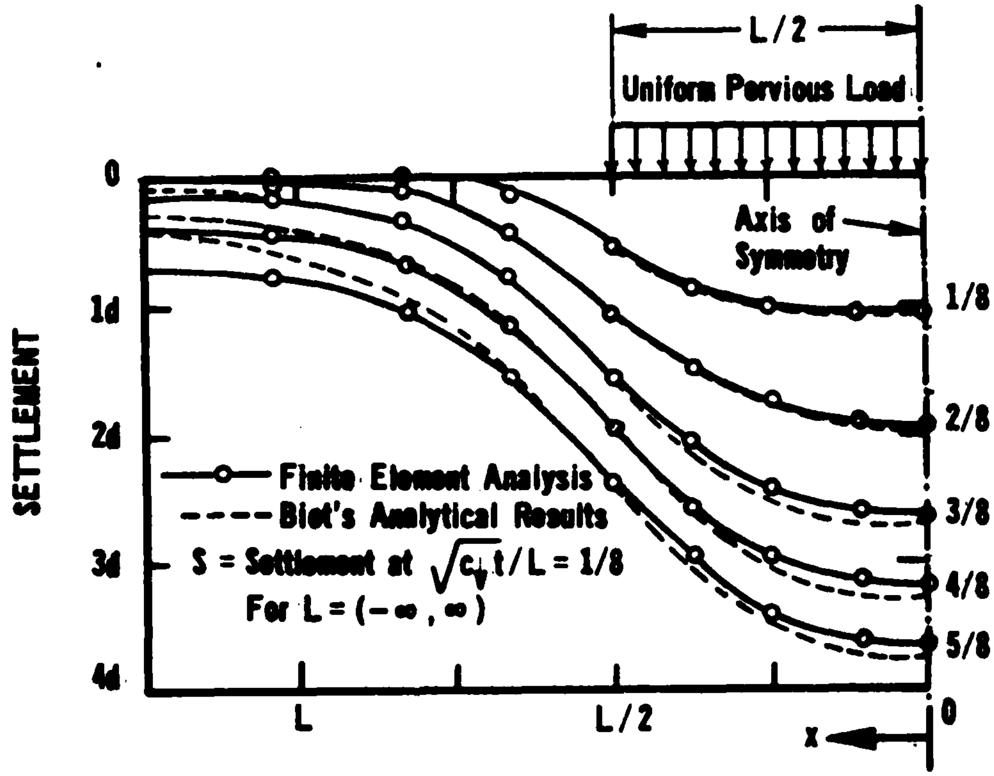

Figure 6. Results of Sandhu and Wilson's Model [6]
Biot's Problem – Rectangular Strip Load

A later work by Ghaboussi and Wilson [8] will be discussed in a subsequent section.

In the discussion of Biot's generalization of Terzaghi's consolidation model, it was noted that, under some circumstances, the heat or diffusion equation can be used to model soil consolidation. Christian and Boehmer [7] developed a displacement-based finite element model for soil consolidation, and compared results for the finite element analyses with a consolidation analysis based on the diffusion equation. Recall that the diffusion form of the consolidation equation (appropriate when the applied load is constant over time) can be written:

$$\dot{p} = c_v \, p,_{jj} \qquad (c_v = \frac{k}{\gamma_w m_v})$$

The volume compressibility m, however, is not a material constant. It depends on the type of analysis. For instance, in the one-dimensional case, the volumetric strain is simply the vertical strain ϵ_z, so m_v is the reciprocal of the constrained elastic modulus ($\epsilon_x = \epsilon_y = 0$). However, in three dimensions, the volumetric strain is $\epsilon_v = \epsilon_x + \epsilon_y + \epsilon_z$, and the value of m_v (and hence c_v) should be modified accordingly. Christian and Boehmer derived correct expressions for the consolidation coefficient c_v and total stress τ for an isotropic soil mass for one, two, and three-dimensional cases (see Table I). With these definitions for c and τ, Biot's consolidation theory can be summarized in the general equation:

$$c_v \, p,_{jj} = \dot{p} - \dot{\tau} \tag{13}$$

(Note that if the applied load is constant over time, $\dot{\tau} = 0$, and Eq. (13) becomes the diffusion equation).

Christian and Boehmer formulated the consolidation problem so the only unknowns were the displacements u. Because of this simplification, the near-

Table 1		
Dimension (1)	τ (2)	c_v (3)
One	σ_z	$\frac{\kappa}{\gamma_w} \frac{\bar{E}(1 - \bar{\nu})}{(1 + \bar{\nu})(1 - 2\bar{\nu})}$
Two	$\frac{\sigma_x + \sigma_y}{2}$	$\frac{\kappa}{\gamma_w} \frac{\bar{E}}{2(1 + \bar{\nu})(1 - 2\bar{\nu})}$
Three	$\frac{\sigma_x + \sigma_y + \sigma_z}{3}$	$\frac{\kappa}{\gamma_w} \frac{\bar{E}}{3(1 - 2\bar{\nu})}$

incompressibility of the soil-water system causes near-infinite terms to occur in the equation set (see Zienkewicz [10] for an explanation of this phenomenon). In order to remove this problem, Christian and Boehmer re-introduce the continuity condition in terms of pore pressures and volumetric strain into the equation set.

Christian and Boehmer compared their finite element model with experimental data from consolidation tests in a triaxial load apparatus. Their results agreed well with experimental and analog solutions. One surprising type of behavior was discovered using the finite element model: pore pressures locally increased with time for some of the triaxial tests. Although this effect was short-lived, it was important because the approximate diffusion equation solution cannot model this phenomenon. Apparently the average consolidation can increase (a global effect) while some parts of the sample experience a decrease in effective stress (a local increase in pore pressure).

The most useful results of this study by Christian and Boehmer were twofold: first, they showed that, except for early stages of consolidation, diffusion solutions can be utilized when the problem is simple enough to make such a solution meaningful. Second, they derived correct expressions for the appropriate volume compressibility m_v, depending upon the geometry of the analysis. These values of m_v insure that, if a diffusion solution is used, it will be one that is appropriate to the problem being analyzed.

However, the finite element model proposed by Christian and Boehmer, because it did not directly model the pore pressure, required some manipulation in order to handle incompressibility of the water-soil system and certain types of flow boundary conditions.

The finite element model proposed in 1971 by Yokoo [9], et al. is practially identical with that proposed by Sandhu and Wilson [6] in 1969 (the latter paper

was discussed earlier in this review). Yokoo's work was completely independent of Sandhu and Wilson, and was more general than the earlier work. In addition, the development of the finite element equations was presented in a more lucid form by Yokoo, who carefully considered admissibility and boundary conditions in the analysis. Another difference between the two models is that Yokoo evaluated the convolution integrals that account for the time marching by using a step-by-step method originally derived by Zienkiewicz and Parekh [19].

Yokoo considered two examples: the 'classic' one-dimensional problem solvable using Terzaghi's theory, and a more practical axisymmetric problem. In the one-dimensional problem Fig. 7, excellent agreement between the finite element model and Terzaghi's diffusion solution was obtained.

The axisymmetric problem modelled by Yokoo was that of a uniform load on a circular plate loading a uniform clay layer. The clay layer exhibits both structural and hydraulic anisotropy. In addition, the applied load is not constant over time. This problem is of interest because it is a useful approximation to a common foundation problem and many of these characteristics (the local distribution of load, anisotropy, and time-dependent loading) violate the assumptions of Terzaghi's theory. Yokoo's paper also contains excellent graphical interpretations of the evolution of pore water pressure, and of the displacement of the soil layer.

Perhaps the greatest advantage that the finite element method has over classical (analytic) solutions for consolidation is that irregular geometry causes no serious difficulties in a finite element analysis. This is particularly important in Geotechnical Engineering, because soil deposits are generally irregular, nonhomogeneous and anisotropic.

Desai and Saxena [10] take advantage of this powerful feature of the finite element method and model consolidation of a layered soil deposit underlying

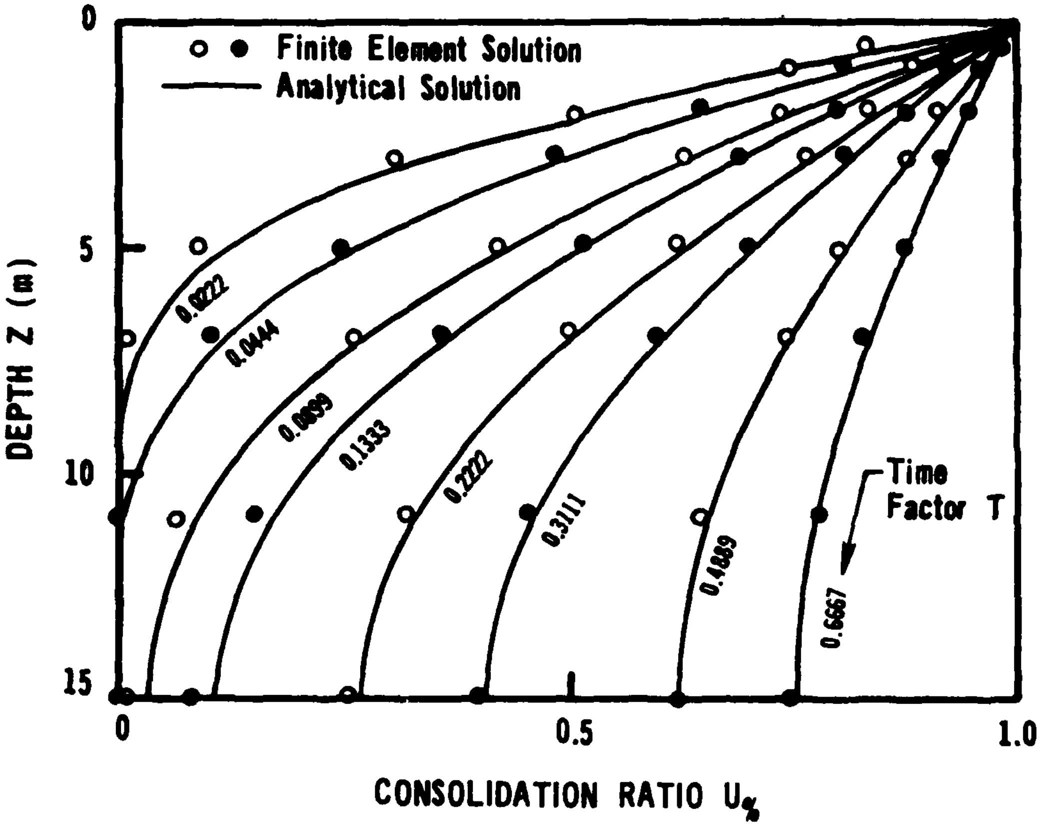

Figure 7. One Dimensional Consolidation From Yokoo's Model [9]

a three building system. The actual geometry of the building site is shown in Fig. 8.

Desai and Saxena's analysis, like the others considered earlier, is based on Biot's consolidation theory. Their model uses a mixed interpolation scheme, with linear interpolation for pore pressure and quadratic approximations for displacement. A centered-difference (Crank-Nicolson) time stepping procedure is used to numerically integrate the convolution integrals that account for the temporal dependence of the variational terms. A useful feature of their model is the ability to vary the length of the time step: since consolidation solutions exhibit behavior resembling exponential decay, the time steps can be lengthened as the solution asymptotically approaches the steady-state solution. (The particular problem is modelled over a period of almost twelve years, with time steps ranging from one to forty days.)

Because soil deposits are heterogeneous, the determination in geotechnical engineering of material parameters often becomes a statistical exercize. For this reason, Desai and Saxena analyzed several models of the three-building foundation problem, each with slightly different material properties and/or degree of anisotropy. The results they obtain are very interesting, Fig. 9.

Five of the six finite element analyses can be seen to closely approximate the actual measured consolidation settlement (shown for building 2). The one analysis that gives a poor result has an unlikely type of anisotropy, where the vertical permeability exceeds the horizontal permeability by a factor of one hundred (the horizontal permeability is generally larger). Desai and Saxena also calculate an estimate for the probable ultimate settlement of this building, using Terzaghi's one-dimensional consolidation theory. Although this theory clearly does not apply here, it is extensively used in similar cases to obtain settlement

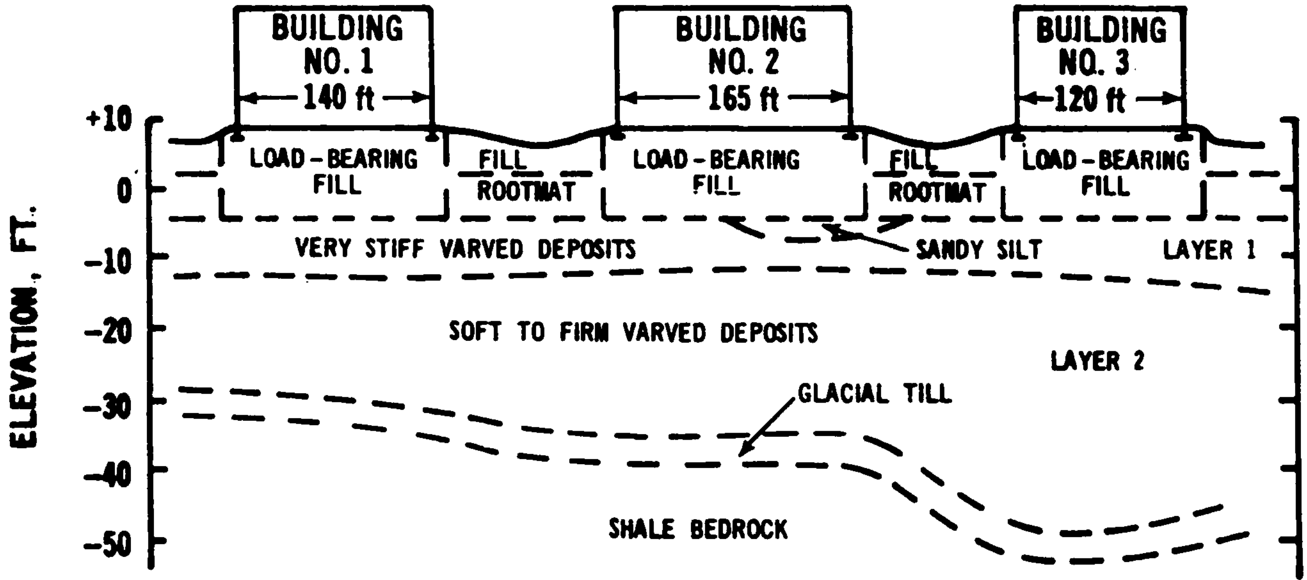

Figure 8. Geometry of Building Site Modelled [10]

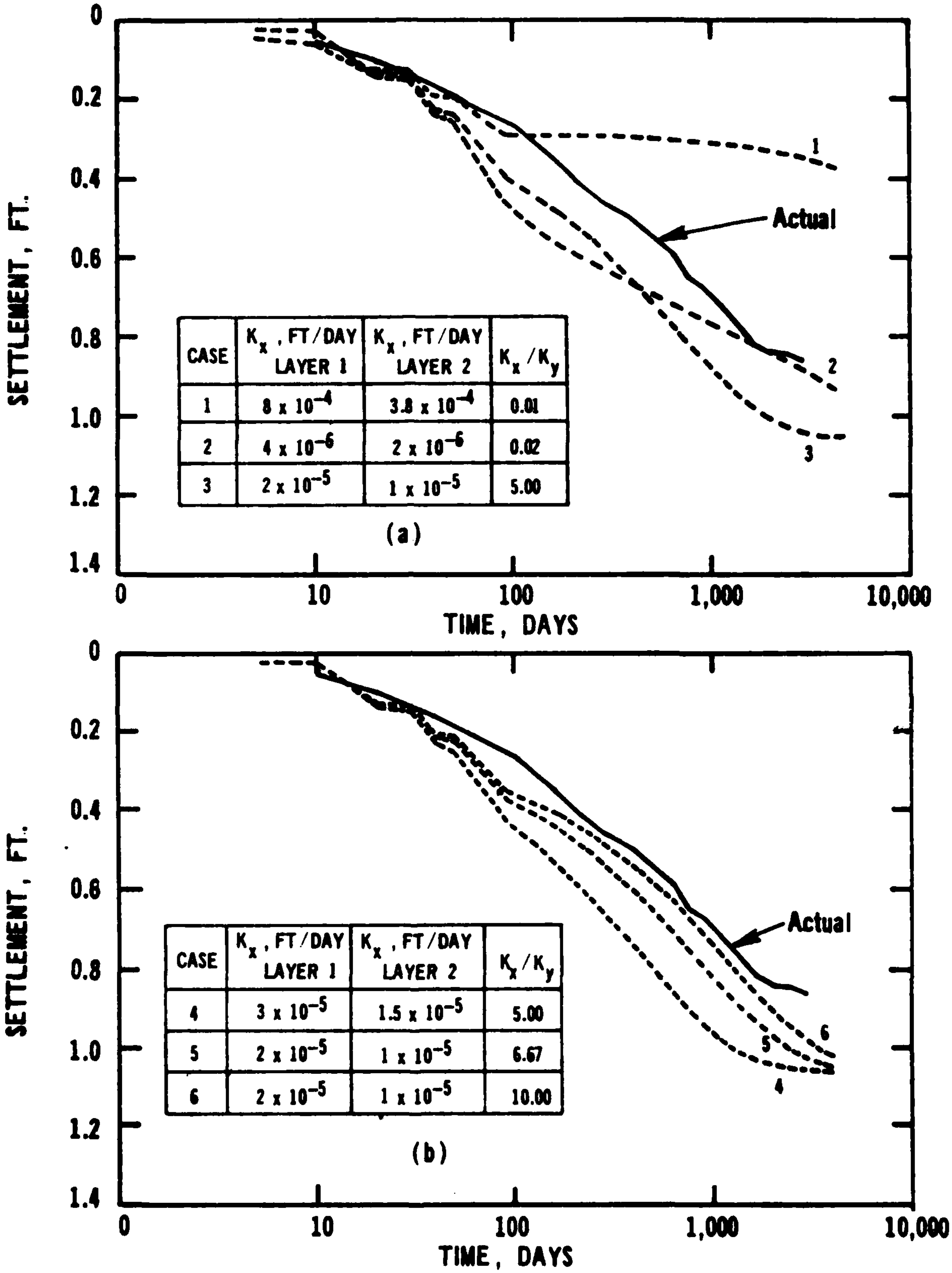

Figure 9. Settlement of Building 2 [10]

estimates: these estimates, in general, are very conservative. However, in this case, actual settlements are 250% of those predicted by Terzaghi's theory!

Desai and Saxena's work is characterized by a practical emphasis, both in terms of the useful problems solved, and the rules of thumb proposed for future analyses. With this work, the advantages of a finite element analysis for consolidation are more fully realized.

Three of the assumptions inherent in Terzaghi's original consolidation theory are sufficiently unrealistic to warrant development of a better model:

A. The soil skeleton is a linear elastic material (i.e. the volume compressibility m_v is constant)

B. Vertical Flow (one-dimensional behavior)

C. Applied (total) stresses are constant over time.

The various finite element models discussed, based on Biot's consolidation theory, have removed the restrictions due to the second and third assumptions. However, none of them have attempted to model the soil skeleton as an inelastic material. Thus the area of consolidation-related research of greatest current interest involves the inclusion of inelastic soil effects. In addition to the work reported in the next section, the reader is referred to [11, 12, and 20].

4. SUMMARY OF GOVERNING EQUATIONS USED IN THIS WORK

For the sake of completeness many of the concepts and equations given in the previous section are repeated here. With only minor changes in notation, sign convention and theory the following is taken from reference [8]; the significant difference is the use of bounding surface plasticity theory to model soil behavior.

Throughout the following sections the usual convention is used that free indices can vary over their ranges and repeated indices must be summed over their ranges; commas denote differentiation.

4.1 Concept of Stress

The pore water pressure, total (phenomological) stress, and effective stress are denoted respectively as h, τ_{ij}, and σ'_{ij}. Pore water pressure is taken to be positive in compression while the mechanics sign convention of tensile normal stresses being positive is used for τ_{ij} and σ'_{ij}. For the purposes of the theoretical development h is taken to be the total pore water pressure (i.e., including the hydrostatic pressure). Later in the discussion of the finite element programs, means for treating it either as total or "excess" pressure are discussed. the effective stress σ'_{ij} is that portion of the total stress carried by the soil skeleton.

The relation between these three stress quantities is [8]:

$$\tau_{ij} = \sigma'_{ij} - \alpha h \delta_{ij} \tag{14}$$

Not all authors include the factor α in the above equation. It appears that its presence permits consideration of the fact that the average stress contribution, over a unit cell of soil, due to the pore pore water pressure may be less than the actual pressure. For example, consider the idealized case illustrated in Fig. 10. However, for actual cohesive soils there is very little stress transfer through particle contact, thus in practise α should be very nearly unity. In fact most researchers [6,9,10] do not include the quantity and Ghaboussi, et al. [8] set it equal to unity for all examples considered. Finally if α is included in this expression and one wishes, for computational purposes, to express the governing equations in variational form it must also be included in the conservation of mass equation. Its physical significance in this second equation is not clear and its inclusion would appear to be arbitrary. Thus equation (14) is rewritten with α set equal to unity.

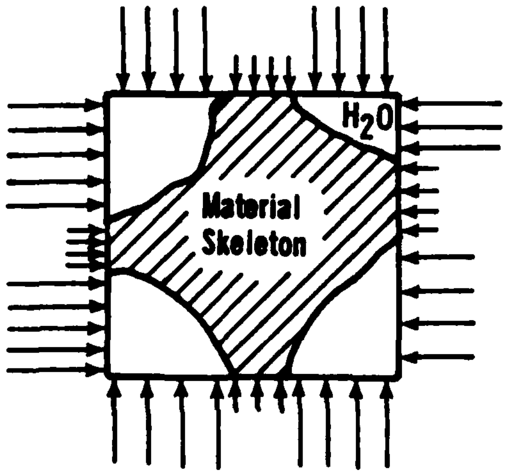

Figure 10. Composite Material For Which $\alpha < 1$

$$\tau_{ij} = \sigma'_{ij} - h\delta_{ij} \tag{15}$$

4.2 Conservation of Mass

Denote the total volume of water that has flowed out of a unit volume of soil by V_w, the strain of the soil mass by ε_{ij} (tensile strain positive) and the total displacement of the soil by u_i. For small strains

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \tag{16}$$

It is assumed that the soil particles and the pore water experience an elastic decrease in volume due to an increase in pore water pressure; denoting the corresponding bulk modulus by Γ, the resulting rate of volume change is $\dot{h}/\Gamma$. (Alternatively the parameter Γ can be thought of as a "penalty number" used to approximately impose an incompressibility constraint on the water and soil particles.) This expression embeds the assumption that the mean stress component of the effective stress produces no significant volume change of the soil particles.

For a completely saturated system the rate of volume change of the soil ($\dot{\varepsilon}_{ii}$) most be balanced by the water flowing out and the rate of volume change of the water and soil particles, i.e.

$$\dot{\varepsilon}_{ii} = -\dot{V}_w - \frac{\dot{h}}{\Gamma} \tag{17}$$

In [8] the factor α of equation (14) multiplies the $\dot{\varepsilon}_{ii}$ term in eq. (17). The presence of α in eq. (17) is necessary if it appears in eq. (14) and it is desired to represent the problem by a variational statement; however, its physical meaning in eq. (17) is unclear.

4.3 Water Flow

An average displacement w_i of the fluid relative to the soil is defined such that $w_i n_i dA$ is the total amount of fluid that crosses the area dA (outward normal n_i). Thus the average (Darcy) velocity of the fluid is

$$v_i = \dot{w}_i \tag{18}$$

Of course, the actual velocity in the pores of the soil is much higher. Denote the effective permeability tensor of the soil, the fluid density and component of gravity in the i direction as k^*_{ij}, ρ_f and g_i respectively. The term effective permeability, as used here, is equal to the ratio of the permeability coefficient commonly used in civil engineering literature divided by the unit weight of water, which in turn is equal to the geometric permeability used in physics divided by the viscosity of the water. In terms of this notation, Darcy's law governing the water flow is expressed as

$$\dot{w}_i = v_i = -k^*_{ij}(h_{,j} - \rho_f g_j) \tag{19}$$

For this application it is assumed that the effective permeability tensor is a constant (i.e., its components are not strain dependent) and symmetric.

4.4 Equilibrium

Denoting the total material density ([illegible] soil) by ρ, the linear equilibrium equations take on their usual [illegible]

$$\tau_{ij,i} + \rho g_j = 0 \tag{20}$$

4.5 Strain - Effective Stress Relations

Soils are in general nonlinear, inelastic [illegible]ence, the stress strain law will involve some type of hereditary relationship. The implementation of

such a relationship usually requires a step-by-step (incremental) solution. For a given time step N, this relationship can be expressed in the form (for convenience the increment number N will not be displayed but merely implied).

$$\Delta\sigma'_{ij} = D_{ijk\ell}\,\Delta\epsilon_{k\ell} + \Delta\sigma'_{ij_o} \tag{21}$$

For actual use in a finite element analysis this equation would be written in matrix form [2,21] however, for the purposes of equation development the tensor notation is preferable. In general the tensor of incremental properties $D_{ijk\ell}$ is a function of the solution (i.e., $\Delta\sigma'_{ij}$ and $\Delta\epsilon_{k\ell}$), and thus some nonlinear solution scheme employing iteration is usually necessary. In many cases the term $\Delta\sigma'_{ij_o}$ is dependent only upon the past history $(0 - t_{N-1})$ of the solution, however, in general it may also depend upon $\Delta\sigma'_{ij}$ and $\Delta\epsilon_{k\ell}$. For the current bounding surface application it is zero. Equation (20) is sufficiently general to accomodate the model of interest in this study, the bounding surface model for cohesive soil, as well as linear elasticity and most other standard and advanced models. For the actual functional form of $D_{ijk\ell}$ for bounding surface theory the reader is referred to ref. [2,22].

4.6 Boundary Conditions

For the sake of brevity only simple boundary condition are considered at this time (i.e. spring and convection type conditions are excluded). At every point on the boundary of the soil mass the rates of either the traction or displacement components will be given, i.e.

$$\dot{t}_j = \dot{\tau}_{ij}\, n_i \text{ or } \dot{u}_j \text{ given} \tag{22}$$

The components of the unit normal to the surface are n_i. In addition the rate of the pore water pressure "h" or the total flow of water ($Q = w_i n_i$) will be known, i.e.

$$\dot{h} \text{ or } \dot{Q} \text{ given} \tag{23}$$

4.7 Incremental Equations

Because of the time dependent nature of the problem a step-by-step solution scheme is required. Thus the variables are [illegible] expressed in an incremental form, i.e. for time N $u_{iN} = u_{iN-1} + \Delta u_i$, etc. [illegible] previously noted the subscripts N on the incremental values are not [illegible] displayed. It is now necessary to express equations (15) - (23) in incremental [illegible]

Consider first eq. (17). Recall that V_w denotes the total volume of water that has flowed out of a unit volume of soil; the rate of this quantity is simply related to the average fluid velocity, i.e.

$$\dot{V}_w = v_{i,i} = \dot{w}_{i,i} \tag{24}$$

Substituting this expression into eq. (17) gives

$$\dot{w}_{i,i} = - \dot{\varepsilon}_{ii} - \frac{\dot{h}}{\Gamma} \tag{25}$$

Integrating the above equation over [illegible] yields (if Γ is not a constant then some form of [illegible] would be required for the last term):

$$\Delta w_{i,i} = - \Delta\varepsilon_{ii} - \frac{\Delta h}{\Gamma} \tag{26}$$

(If one prefers the above step can be [illegible] setting a weighted residual of eq. (25) to zero.)

Integrating eq. (19) over the time interval gives:

$$\Delta w_i = - \int_{t_{N-1}}^{t_N} k^*_{ij} \, (h,_j - \rho_f g_j) \, dt \tag{27}$$

It is assumed that k^*_{ij} is a constant (permitting it to be state dependent, hence implicitly time dependent, would only slightly complicate the analysis), i.e.

$$\Delta w_i = - k^*_{ij} \left\{ \int_{t_{N-1}}^{t_N} h,_j \, dt + \int_{t_{N-1}}^{t_N} \rho_f g_j dt \right\} \tag{28}$$

In order to accomodate possible centrifuge applications the gravity term is considered to be time dependent. The two integrals on the right are now approximated using numerical integration. Trapezoidal integration is used on the second term, while the more general rule

$$\int_{t_{N-1}}^{t_N} F(t)dt \simeq [(1-\theta) \, F_{N-1} + \theta F_N]\Delta t = [F_{N-1} + \theta \Delta F]\Delta t \tag{29}$$

is used for the first term. Thus, eq. (28) yields $(\overline{\rho_f g_j} = \frac{1}{2} [(\rho_f g_j)_{N-1} + (\rho_f g_j)_N])$:

$$\Delta w_i \simeq - k^*_{ij} [h_{N-1,j} + \theta \Delta h,_j - \overline{\rho_f g_j}]\Delta t \tag{30}$$

Values of θ of 0, 1 and 1/2 give forward integration, backwards integration and the trapezoidal rule respectively; alternatively if one prefers to discretize time by approximating the time derivative in eq. (19) using a finite difference operator the cited values of θ correspond to using a forward difference (Euler's method), a backward difference and a central difference (Crank-Nicolson or mid-point method); finally if one prefers the weighted residual interpretation [18] these values of θ correspond to a delta function weight at the backward point, a delta function weight at the forward point and a uniform weight respectively. Values

of $\theta \leq 1/2$ give schemes which are only conditionally stable and should be avoided. Theoretically $\theta = 1/2$ should lead to the greatest accuracy, but practically may lead to oscillation problems. Theoretically, a value of $\theta = 1$ eliminates all oscillati[illegible] but [illegible] poor convergence characteristics. Zienkiewicz [18] suggests a compromise [illegible] which [illegible] calls "Galerkin's" method as it bears a resemblance to Galerkin's weighted residual method for boundary value problems. The unlimited possibilities offered by higher-order integration formulas (e.g., improved trapezoidal etc.) are not explored here.

The incremental forms of eqns. (15), [illegible] and (20) are found simply by writing the equations at t_{N-1} and t_N and subtracting. Equations (22) and (23) are converted to incremental form by [illegible] the interval. The results along with eqs. (21) (26) and (30) are summarized below (eqs. (15) and (21) are combined):

Field equations:

$$\Delta\tau_{ij} = D_{ijk\ell}\,\Delta\varepsilon_{k\ell} \; \Delta u \delta_{ij} \; [illegible] \tag{31}$$

$$\Delta w_i = -k^*_{ij}[h_{N-1,j} + \theta\Delta h_{,j} \; [illegible]]\Delta t \tag{32}$$

$$\Delta\varepsilon_{ij} = \frac{1}{2}(\Delta u_{i,j} + \Delta u_{j,i}) \tag{33}$$

$$\Delta\tau_{ij,i} + \Delta(\rho g_j) = 0 \tag{34}$$

$$\Delta w_{i,i} + \Delta\varepsilon_{ii} + \frac{\Delta}{\Gamma} \; [illegible] \tag{35}$$

boundary conditions:

$$\Delta T_j = \Delta\tau_{ij}\, n_i \text{ or } [illegible] \tag{36}$$

and

ΔQ or Δh given (37)

This set of equations can be reduced in number by expressing them in terms of primary dependent variables Δu_i and Δh. Equation (33) is substituted into eq. (31) and the results into eq. (34), and eqs. (32) and (33) are substituted into eq. (35) to yield:

$$(D_{ijk\ell}\ \Delta u_{k,\ell})_{,i} - \Delta h_{,i}\ \delta_{ij} + \Delta\sigma'_{ij_{o,i}} + \Delta(\rho g_j) = 0 \qquad (38)$$

and

$$\{k^*_{ij}[h_{N-1,_j} + \theta\Delta h_{,j} - \overline{\rho_f g_j}]\}_{,i}\ \Delta t - \Delta u_{i,i} - \frac{\Delta h}{\Gamma} = 0 \qquad (39)$$

The corresponding boundary conditions are found by substituting eqs. (31), (32) and (33) into eqs. (36) and (37):

$$\Delta T_j = (D_{ijk\ell}\ \Delta u_{k,\ell} - \Delta h\ \delta_{ij} + \Delta\sigma'_{ij_o})n_i \text{ or } \Delta u_j \text{ given} \qquad (40)$$

and

$$\Delta Q = -\ k^*_{ij}[h_{N-1,_j} + \theta\Delta h_{,j} - \overline{\rho_f g_j}]n_i\ \Delta t \text{ or } \Delta h \text{ given} \qquad (41)$$

Thus eqs. (38) and (39) are the final form of the governing incremental equations (for time step N), subject to the boundary conditions of eqs. (40) and (41).

4.8 Variational Statement of the Problem

The solution of the boundary value problem given by eqs. (38) - (41) will, except in the simplest cases, require numerical analysis. A finite element solution of these equations can be formulated directly by applying Galerkin's

weighted residual method or alternatively by seeking a stationary point for an appropriate variational statement of the problem; the latter method is used.

Using variational calculus concepts it is a simple matter to construct a variational statement that is equivalent to these equations; it has the following form:

$$\delta F = 0 \tag{42}$$

where

$$\begin{aligned} F = \iiint_V \{ & \tfrac{1}{2}\, \Delta\varepsilon_{ij}\, D_{ijk\ell}\, \Delta\varepsilon_{k\ell} - [\text{illegible}] \ \frac{1}{2\Gamma}(\Delta h)^2 \\ & - \theta \frac{\Delta t}{2}\, k^*_{ij}\, \Delta h_{,j}\, \Delta h [\text{illegible}]_{ij} \\ & - \Delta t\, k^*_{ij}(h_{N-1,j} - \rho_f g [\text{illegible}]\, g_i)\, \Delta u_i \} dV \\ & - \int_{S_1} \Delta T_i\, \Delta u_i\, dS - \int_{S_2} \Delta Q\, \Delta h [\text{illegible}] \end{aligned} \tag{43}$$

The volume of the soil mass is denoted by V and the surface areas over which ΔT_i and ΔQ are specified, by S_1 and S_2. (An alternative course of action is to bypass the incremental differential equations and to obtain eq. (43) directly from a variational statement of the type given in eq. (12) and a numerical approximation to the convolution integrals; this approach would, however, require a somewhat different numerical [illegible] bounding surface plasticity model.)

5. FINITE ELEMENT ANALYSIS

For the two- and three-dimensional [illegible] analyses developed as part of this project, standard [illegible] parametric elements are

used. This selection was based on a consideration of the ease of data preparation and to a lesser extent on the results of an informal report by Professor Segerlind of Michigan State which indicates that for time dependent problems the low order elements experience fewer oscillation problems than the higher-order ones. Because the steps required to proceed from eq. (42) to the finite element equations are well documented (e.g., see [18]) only a few special considerations are discussed here.

For a linear elastic material $D_{ijk\ell}$ is constant and the analysis proceeds simply in a step-by-step fashion. However, for a cohesive soil characterized by the bounding surface plasticity model, this tensor is highly dependent upon the solution and hence each incremental analysis is decidedly nonlinear. The approximate Newton-Raphson method used to solve the nonlinear problem is throughly discussed in [23]. The two programs are written so that a user specified value of β between 0 and .5 is used to select an approximate Newton-Raphson method on the spectrum from "tangent stiffness" to "successive approximations" [23,24,25].

The one characteristic of the problem that requires some care is the handling of the near-incompressibility of the soil when it is in a saturated condition. The most general procedure for avoiding the accuracy and round-off error problems associated with the finite element analysis of nearly incompressible materials is the use of a "mixed formulation" analysis [19,26]. Its use is natural for soil consolidation problems because the additional mean pressure variable, needed in the mixed formulation, is already included in order to describe the flow problem, i.e., eq. (42) is a natural "mixed" statement of the problem.

The use of the mixed formulation for isoparametric elements must, however, be done with considerable care. The problem is, if the near-incompressibility condition is applied point-wise, the elements "lock-up", i.e.,

become rigid and will not deform [18]. To avoid this problem, the near-incompressibility condition must not be satisfied point-wise but only in some average sense. For a first order element only the average volume change for the element can be made zero. This is accomplished if the term in eq. (43) which measures the volume change, measures only average volume change (over the element) and not point-wise change; the term in question is $\Delta h \; \Delta\epsilon_{ii}$. In order to measure only average volume change, the element approximation for Δh in this term must be a constant. However, the admissibility condition for Δh which arises out of the presence of the term $\Delta h,_j \; \Delta h,_i$ requires that its approximation, for this second term, be continuous between elements. This incongruency can be easily dealt with by using a two field approximation for Δh. The first (used in the term $\Delta h \; \Delta\epsilon_{ii}$) is constant for each element and thus not continuous across element boundaries, the second is continuous and is used for all other terms in eq. (43). The two fields are related to a common set of nodal unknowns. The continuous field is defined by the node point values and first order, isoparametric shape functions. The constant element value for the discontinuous field is defined to be the average of the values for the nodes describing the element, thus

$$\Delta h_{(1)} = \frac{1}{ND} \Delta H_i \; I_i \tag{44}$$

$$\Delta h_{(2)} = \Delta H_i \; N_i \tag{45}$$

Where ND is the number of nodes defining the element (4 or 8), N_i are the first order isoparametric shape functions, ΔH_i are the node point values of Δh for the nodes defining the element and $I_i = 1$. When one prefers to use the continuous approximation for Δh in all terms then I_i is replaced by N_i. For the programs written to evaluate the analysis, the user can choose between these two alternatives by means of a simple input code.

The isoparametric-Laplacian grid generation scheme given in [27] has been modified in order to replace the iterative solution by a direct solution; the reduction in computational cost for this step in the analysis is dramatic. For the three-dimensional code, this grid generation scheme has been generalized to produce meshes consisting of 8-node brick elements.

For unsaturated or "ideal drained" conditions the h variable is dropped to reduce the number of unknown's per node by one. However, for "ideal undrained" conditions and a saturated soil, the h variable is retained to facilitate modeling the resulting near-incompressibility (by means of the mixed formulation as explained above).

The implementation of the bounding surface model followed directly the instructions given in [22]. Reference [22] is a revision of a portion of reference [2] to reflect a complete recoding of subroutine CLAY and its affiliated subroutines. CLAY was recoded for the sake of clarity, to incorporate some recent minor changes in the model, to improve numerical efficiency and to take advantage of the structured programming concepts of "Fortran 77." For the sake of illustrating the implementation procedure (according to the well-documented instructions in [22]) of the bounding surface model in finite element codes, no changes whatsoever were made in CLAY and its subroutines for this application. As a result one unused subroutine is retained and there is duplicate input for the quantity Γ. In order to simulate the modification of existing programs, the two finite element programs developed for this project were first written as relatively general incremental-iterative nonlinear programs and then the CLAY subroutine was included (by means of two simple call statements per program) as a modular unit.

The notation used in the following discussion of this implementation is the same as used in [22]. Subroutine RPROP is called from subroutine PROPTY

and reads the parameters which describe the bounding surface model. For convenience the combined bulk modulus of the soil particles and pore water Γ (not really a parameter of the plasticity model) is read and stored separately by PROPTY; the read for Γ in RPROP is a duplication and is not necessary. The values of void ratio e_o and preconsolidation pressure p_o in array STOR are initialized for each element in Subroutine GEOM. The STOR array for each element is included in the "BLK6" records stored on unit 2.

The analysis is a mixed formulation (see Section 5) and thus in the CALL to CLAY (from PROPTY) KIND = 0. The finite element program supplies h_{N-1} and Δh_N to CLAY through the "CALL". Because in this application it was found convient to store Γ in the main program, the quantity GAM in the CALL is not used. The combining of the arrays $[D]_{N-1,K-1}$ and $[D]_{N,K-1}$ (according to eqs. (17) and (18) of [22]) is done in PROPTY immediately after the CALL to CLAY. The reversing of the sign convention for the normal stresses and strains (and for the 2-D program, the expanding of the two-dimensional stress and strain vectors to three-dimensional form) is done just prior to the CALL to CLAY. For this small deformation analysis LARGE = 0. LOCIT is set equal to ITMAX used for the global iteration and ERMAX is set equal to 10 times the value used in the global iteration. TH1 has the value of .5 as used in the main program. In all cases IDIM = 3 (this is true for plane strain conditions as the finite element analysis calculates σ_z). Information concerning use of the programs is to be found in [28] and [29].

Two-dimensional element matrices:

For plane strain conditions to exist, the only off diagonal term in the k^*_{ij} tensor that can be non-zero is $k^*_{xy}(k^*_{rz})$; this coefficient is denoted as k^*_{12}.

Expressing the displacement approximations in terms of their node point values and the shape functions, using eqs. (44) and (45) in the appropriate terms and differentiating with respect to the node point unknowns yields the element matrices. The terms arising because of the presence of the Δh variable are explicitly given below:

$$\frac{\partial F}{\partial \Delta U_i} = \iint_A \{ [\quad]\Delta U_j + [\quad]\Delta V_j + [- \frac{I_j}{4} (F_i + \rho N_i)] \Delta H_j + [\quad] \} RdA$$

$$\frac{\partial F}{\partial \Delta V_i} = \iint_A \{ [\quad]\Delta U_j + [\quad]\Delta V_j + [- \frac{I_j}{4} G_i] \Delta H_j + [\quad] \} RdA$$

$$\frac{\partial F}{\partial \Delta H_i} = \iint_A \{ [- \frac{I_i}{4} (F_j + \rho N_j)] \Delta U_j + [- \frac{I_i}{4} G_j] \Delta V_j$$

$$+ \{- \frac{1}{r} N_i N_j - \theta \Delta t [k^*_{11} F_i F_j + k^*_{12}(F_i G_j + F_j G_i) + k^*_{22} G_i G_j] \} \Delta H_j$$

$$- \Delta t \{ [k^*_{11}(H_{N-1_k} F_k - \rho_f g_1) + k^*_{12}(H_{N-1_k} G_k - \rho_f g_2)] F_i$$

$$+ [k^*_{12}(H_{N-1_k} F_k - \rho_f g_1) + k^*_{22}(H_{N-1_k} G_k - \rho_f g_2)] G_i \} RdA$$

The terms not shown are identical to those for conventional stress analysis. For plane strain conditions $R = 1$, $\rho = 0$, while for axisymmetry $R = r$ and $\rho = \frac{1}{r}$. The x(r) and y(z) derivatives of the shape functions (N_i) are denoted respectively by F_i and G_i.

Three-dimensional element matrices:

The terms in the element matrices arising from the presence of the Δh variable are given below: (Note that ΔW_i is the change in displacement in the z-direction, not the average fluid displacement W_i. Also, the derivatives of the shape functions with respect to the global coordinate directions x, y and z are

The empty brackets indicate terms which are identical to those found in a conventional stress analysis.

6. EXAMPLES

During the check-out phase of the code development numerous example problems were analyzed. Results from two of these analyses are given in Figures 11 and 12.

The first example is a generalization, to include the compressibility of the soil particles and pore water ($\Gamma < \infty$ in eq. (17)), of the one-dimensional Terzaghi problem. In Figure 11 the finite element predictions for the deflection at the surface are compared to the exact results taken from [8]. It should be noted that the interpretation of the results given in [8] must be done with care. The contents of Figure 1 give the impression that the solution only depends on the parameter M/C_v, whereas it is easy to show that it depends instead on the quantity $\frac{Mk}{C_v \eta}$ (the terms are defined in [8]). The results given in Figure 1 of [8] appear to have been run for the case of $k/\eta = 1.0$ and thus the ambiguity caused no problem.

The second example considered the uniform loading of a soil layer that is free to drain both at the surface and into a central sand drain. The finite element mesh used in the analysis is illustrated in [28]. Figure 12 compares the predictions for the surface displacement, at a radius 20 times that of the drain, to the results given in [8].

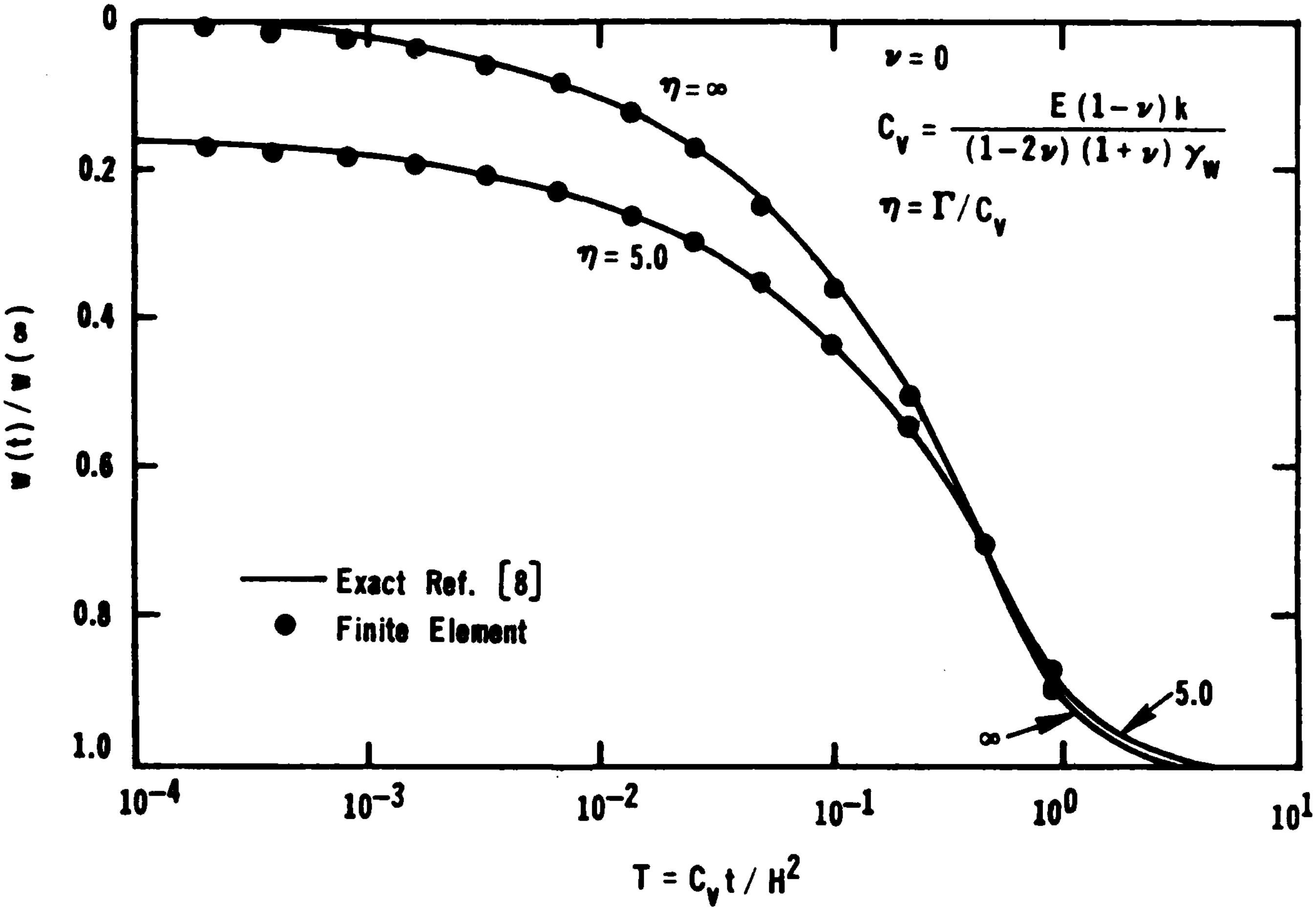

Figure 11. Terzahi's Problem

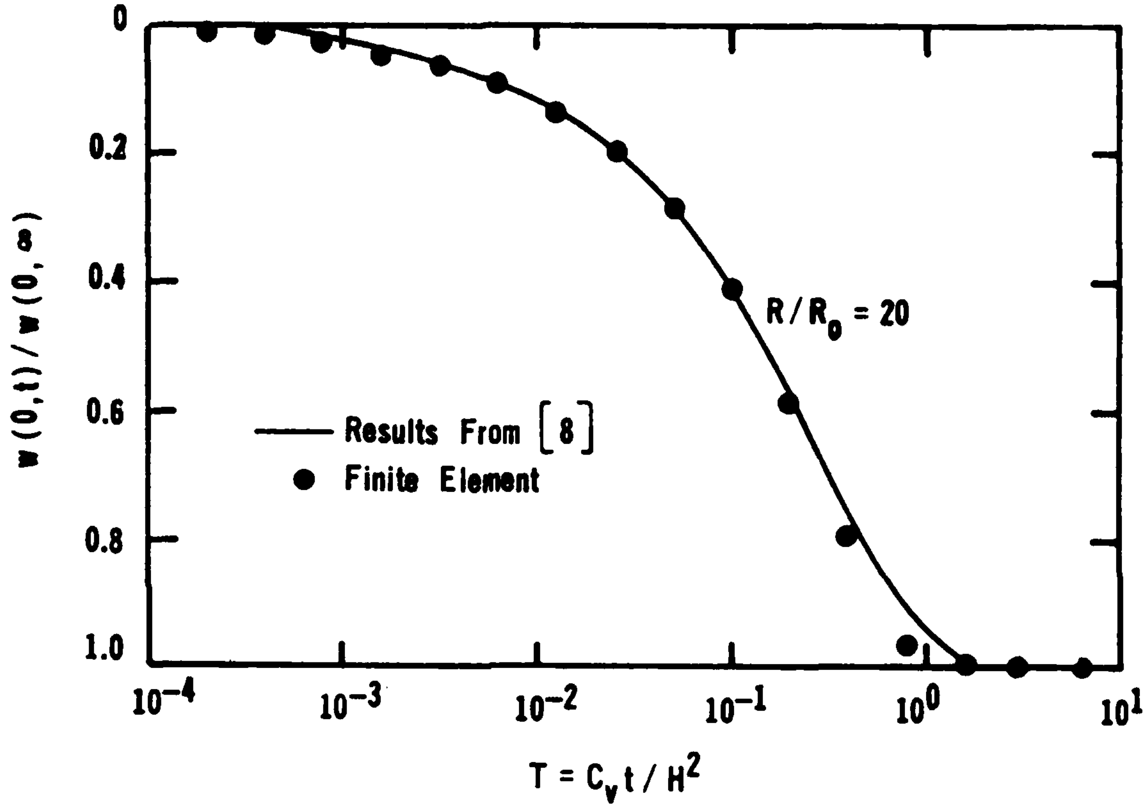

Figure 12. Sand Drain Example

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